

# Multiobjective Krill Herd Algorithm for Electromagnetic Optimization

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Metaheuristics have recently become the forefront of the current research as a powerful way to deal with many electromagnetic optimization problems. Based on the simulation of the herding behavior of krill individuals, a krill herd (KH) algorithm was recently proposed to solve optimization problems. In order to extend the classical mono-objective KH algorithm approach, this paper proposes a new multiobjective KH (MKH) algorithm and a modified MKH approach using the beta distribution in the inertia weight tuning. Numerical results on a multiobjective constrained brushless direct current (DC) motor design problem show that the evaluated MKH algorithms present a promising performance.

**Index Terms**— Optimization, brushless DC motor design, krill herd algorithm, multiobjective optimization.

## I. INTRODUCTION

RECENTLY, Gandomi and Alavi [1] proposed the krill herd (KH) algorithm, which is based on the simulation of the herding behavior of krill individuals in nature. In KH, the objective function for the krill movement is determined by the minimum distances of each individual krill from food and from highest density of the herd. KH has shown promising results when applied to single-objective global optimization problems [2], [3]. Yet, the KH can be extended to solve multiobjective optimization problems (MOPs). Unlike a single-objective optimization problem, a MOP does not, in general, have a unique optimal solution. Instead, the optimal solutions to a MOP constitute possibly an infinite set of compromise solutions, known as Pareto optimal solutions or non-dominated solutions, which can be ordered only by subjective preferences.

By extending the basic ideas for single-objective optimization of KH, a multiobjective KH (MKH) approach and a modified MKH using beta distribution in the inertia weight tuning are proposed in this paper to increase the solutions convergence and the population diversity. A brushless direct current (DC) wheel motor benchmark problem [4], [5] is used to investigate the performance of the MKH and the modified MKH approaches.

## II. FUNDAMENTALS OF THE KRILL HERD APPROACHES

In KH, the time-dependent position of the krill individuals is formulated by three main factors: (i) motion induced, (ii) foraging motion, and (iii) physical diffusion. The motion for a krill individual is induced from other krill. The foraging motion is formulated in terms of two main effective parameters. The first is the food location and the second one is the previous experience about the food location. In KH, the virtual center of food concentration is approximately calculated according to the fitness distribution of the krill individuals, which is inspired from “center of mass”.

In the present work an adaptation of the original KH algorithm as described above is proposed, in order to cope with more than one and possibly conflicting objectives. The aim is to accomplish the goals in multiobjective optimization

such as convergence and approximation to approximate the true Pareto front, as well as diversify its solutions in such way that at the end of each run the engineer has representative solutions for the problem at hand.

Being so, it was employed a truncation procedure as in the NSGA-II (Nondominated Sorting Genetic Algorithm - version II) [4] and the global best individual selection as in MOPSO-CD (Multiobjective Particle Swarm Optimization with Crowding Distance) [5]. In the truncation procedure, at the end of each iteration both the parent and child populations of krill are combined and sorted according to the non-inferiority and crowding distance criteria. The solutions are kept according to this ranking. In order to choose the global best krill, we use the same ranking and choose randomly among the 10% best ranked krill. Thus, krill which explore a nondominated and less populated region are given more chance to influence the other along the iterations of the algorithm. The procedure for implementing the MOKH can be summarized as the pseudo code shown in Fig. 1.

The use of the beta probability distribution [6] can be useful to preserve diversity and helps to explore hidden areas in the search space. In MOKH approach ( $p > 0$ ), the  $p \in [0, 1]$  is related to the percentage  $p$  of the classical update of inertia weights utilization and  $(1-p)$  of the utilization of the beta probability distribution in the inertia weights tuning. The classical update of inertia weights ( $\omega_n$  and  $\omega$ ) adopted both 0.9 at beginning and linearly decreased to 0.1.

```
Generate and evaluate the initial population krill
Initialize the iteration
While the termination criterion iteration < Maxiteration is not satisfied
  For i = 1: (population size of krills) do
    Perform the following motion calculation
    Motion induced by the presence of other individuals
    Foraging motion and physical diffusion
    Implement the genetic operators
    Update the krill individual position in the search space
  End for
  Assign each krill a rank equal to its nondomination level
  Insert nondominated krills into the external archive
  iteration = iteration + 1
EndWhile
Postprocess the optimization results
```

Fig. 1. Pseudo code of the MOKH.

### III. BRUSHLESS DC MOTOR DESIGN

A brushless DC wheel motor benchmark was presented in [7] and the code for computing the objective function is publicly available [8].

The problem is characterized by five continuous design variables (see Table I) subject to six inequality constraints which are related to technological, operational and considerations regarding the wheel motor. Here, the objectives are the minimization of  $f_1=1-\eta$ , where  $\eta$  is the efficiency, and  $f_2=M_{tot}$ , where  $M_{tot}$  is the total mass, which has the constraint  $M_{tot} \leq 15$  kg.

The MOKH approaches must be supplemented with a mechanism to efficiently handle constraints. In this paper, a third objective function  $f_3$  to be minimized related to the number of infeasible constraints are adopted in the optimization procedure.

### IV. NUMERICAL EXPERIMENTS

Different MOKH are compared on the brushless DC motor optimization problem. In all 30 experiments were used the same parameters for MOKH approaches, namely the population size of 30 krill, an external archive of 200 krill, and a stopping criterion of 6,000 function evaluations in each run. Furthermore, it was adopted the foraging speed  $V_f=0.02$ , the maximum induced speed  $N_{max}=0.01$ , and the maximum and minimum diffusion speeds of  $D_{max}=0.01$  and  $D_{min}=0.002$ .

Results for 30 runs are shown in Tables II, III and IV, while Fig. 2 shows the obtained Pareto fronts. According to the simulation results, the MOKH ( $p=0.5$ ) presented promising results in terms of spacing, number of solutions in the Pareto front and Euclidian distances (see results in bold in Table II).

TABLE I  
OPTIMIZATION VARIABLES AND SEARCH RANGE

Variable	Meaning	Minimum value	Maximum value
$D_s$ [m]	Bore (stator) diameter	0.15	0.33
$B_e$ [T]	Air gap induction	0.50	0.76
$\delta$ [A/m <sup>2</sup> ]	Conductor current density	2.0E6	5.0E6
$B_d$ [T]	Teeth magnetic induction	0.9	1.8
$B_{cs}$ [T]	Stator back iron induction	0.6	1.6

### V. CONCLUSION

For MOPs, evolutionary and swarm intelligence algorithms in general have demonstrated to be effective and efficient tools for finding approximations of the Pareto front. In this paper, the MOKH algorithms with different  $p$  values are compared to solve the brushless DC motor benchmark problem. The proposed MOKH approaches provided good results in terms of mean values (30 runs) of spacing and normalized Euclidean distances. Future research will focus on MOKH with mechanisms to hold the diversity of the population when applied to electromagnetic optimization.

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TABLE II  
SPACING AND EUCLIDEAN DISTANCES INDICES (30 RUNS)  
FOR THE FEASIBLE SOLUTIONS ( $f_3 = 0$ ) USING MOKH

$p$	PF*	ED <sup>#</sup>	SP <sup>%</sup>
0	63	1.5934	0.0353
0.1	87	1.3851	0.0243
0.2	73	1.4900	0.0352
0.3	65	1.5630	0.0423
0.4	71	1.5186	0.0298
0.5	<b>97</b>	<b>1.3128</b>	<b>0.0226</b>
0.6	79	1.4485	0.0228
0.7	90	1.3397	0.0191
0.8	80	1.4292	0.0269
0.9	83	1.3884	0.0305
1	83	1.4030	0.0244

\* PF: Pareto front (filtered of 30 runs). # ED: Normalized Euclidean distance ( $f_1, f_2$ ) until the origin. % SP: Normalized spacing between the ( $f_1, f_2$ ) values(.

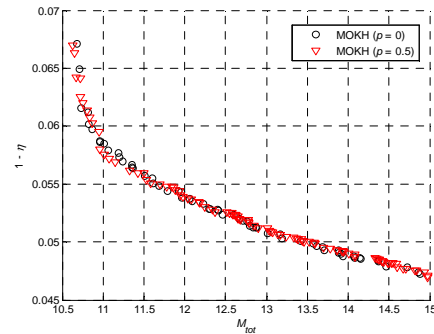


Fig. 2. Pareto set points (filtered of 30 runs) using MOKH approaches.

TABLE III  
RESULTS IN TERMS OF DECISION VARIABLES FOR MOKH ( $p=0$ )

Indices	$D_s$ [m]	$B_e$ [T]	$\delta$ [A/m <sup>2</sup> ]	$B_d$ [T]	$B_{cs}$ [T]
<sup>1</sup> Best $M_{tot}$	0.1919	0.6580	3.9728E6	1.7710	1.5943
<sup>2</sup> Best $1-\eta$	0.2020	0.6599	2.0128E6	1.7995	1.1688
<sup>3</sup> MeanOF	0.1799	0.6734	2.8138E6	1.7980	1.5215

<sup>1</sup>  $M_{tot} = 10.5858$  and  $1-\eta = 0.0551$ ; <sup>2</sup>  $M_{tot} = 14.8804$  and  $1-\eta = 0.0472$ ;  
<sup>3</sup>  $M_{tot} = 11.6188$  and  $1-\eta = 0.0550$ .

TABLE IV  
RESULTS IN TERMS OF DECISION VARIABLES FOR MOKH ( $p=0.5$ )

Indices	$D_s$ [m]	$B_e$ [T]	$\delta$ [A/m <sup>2</sup> ]	$B_d$ [T]	$B_{cs}$ [T]
<sup>1</sup> Best $M_{tot}$	0.1921	0.6521	3.9242E6	1.7987	1.5971
<sup>2</sup> Best $1-\eta$	0.2010	0.6497	2.0135E6	1.7841	0.9979
<sup>3</sup> MeanOF	1.7972	0.6628	2.8029E6	1.7927	1.4588

<sup>1</sup>  $M_{tot} = 10.6297$  and  $1-\eta = 0.06693$ ; <sup>2</sup>  $M_{tot} = 14.9726$  and  $1-\eta = 0.04696$ ;  
<sup>3</sup>  $M_{tot} = 11.5791$  and  $1-\eta = 0.0550$ .